

CYCLIC STRESS STUDIES BY TIME-AVERAGED PHOTOELASTICITY

BY

Y. Y. HUNG, C. Y. LIANG, J. D. HOVANESIAN AND A. J. DURELLI

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CYCLIC STRESS STUDIES BY TIME-AVERAGED PHOTOELASTICITY

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Y. Y. Hung, C. Y. Liang, J. D. Hovanesian and A. J. Durelli

ABSTRACT

This paper describes an experimental method whereby the amplitude of cyclic stresses may be readily determined by time-averaged photoelasticity. Using an ordinary polariscope with a monochromatic light source, "time-averaged isochromatics fringes" are formed if the photographic film in the camera is exposed with an exposure time equal to one or several periods while the photoelastic model is undergoing stready state cyclic loading. The fringe pattern depicts amplitudes of the oscillating stresses according to the zeroth order Bessel function. These properties permit the determination of a time-averaged cyclic stress-optic law. It is also possible to use the method to determine time-averaged isoclinics. The method has great potentiality in the study of in-plane vibrations.

Previous Technical Reports to the Office of Naval Research

- A. J. Durelli, "Development of Experimental Stress Analysis Methods to Determine Stresses and Strains in Solid Propellant Grains"--June 1962. Developments in the manufacturing of grain-propellant models are reported. Two methods are given: a) cementing routed layers and b) casting.
- 2. A. J. Durelli and V. J. Parks, "New Method to Determine Restrained Shrinkage Stresses in Propellant Grain Models"--October 1962. The birefringence exhibited in the curing process of a partially restrained polyurethane rubber is used to determine the stress associated with restrained shrinkage in models of solid propellant grains partially bonded to the case.
- 3. A. J. Durelli, "Recent Advances in the Application of Photoelasticity in the Missile Industry"--October 1962.

 Two- and three-dimensional photoelastic analysis of grains loaded by pressure and by temperature are presented. Some applications to the optimization of fillet contours and to the redesign of case joints are also included.
- 4. A. J. Durelli and V. J. Parks, "Experimental Solution of Some Mixed Boundary Value Problems"--April 1964.

 Means of applying known displacements and known stresses to the boundaries of models used in experimental stress analysis are given. The application of some of these methods to the analysis of stresses in the field of solid propellant grains is illustrated. The presence of the "pinching effect" is discussed.
- 5. A. J. Durelli, "Brief Review of the State of the Art and Expected Advance in Experimental Stress and Strain Analysis of Solid Propellant Grains"--April 1964. A brief review is made of the state of the experimental stress and strain analysis of solid propellant grains. A discussion of the prospects for the next fifteen years is added.
- 6. A. J. Durelli, "Experimental Strain and Stress Analysis of Solid Propellant Rocket Motors"--March 1965.

 A review is made of the experimental methods used to strain-analyze solid propellant rocket motor shells and grains when subjected to different loading conditions. Methods directed at the determination of strains in actual rockets are included.
- 7. L. Ferrer, V. J. Parks and A. J. Durelli, "An Experimental Method to Analyze Gravitational Stresses in Two-Dimensional Problems"--October 1965.

 Photoelasticity and moiré methods are used to solve two-dimensional problems in which gravity-stresses are present.

- 8. A. J. Durelli, V. J. Parks and C. J. del Rio, "Stresses in a Square Slab Bonded on One Face to a Rigid Plate and Shrunk"--November 1965.

 A square epoxy slab was bonded to a rigid plate on one of its faces in the process of curing. In the same process the photoelastic effects associated with a state of restrained shrinkage were "frozen-in." Three-dimensional photoelasticity was used in the analysis.
- 9. A. J. Durelli, V. J. Parks and C. J. del Rio, "Experimental Determination of Stresses and Displacements in Thick-Wall Cylinders of Complicated Shape"--April 1966.

 Photoelasticity and moiré are used to analyze a three-dimensional rocket shape with a star shaped core subjected to internal pressure.
- 10. V. J. Parks, A. J. Durelli and L. Ferrer, "Gravitational Stresses Determined Using Immersion Techniques"--July 1966.

 The methods presented in Technical Report No. 7 above are extended to three-dimensions. Immersion is used to increase response.
- 11. A. J. Durelli and V. J. Parks. "Experimental Stress Analysis of Loaded Boundaries in Two-Dimensional Second Boundary Value Problems"-- February 1967.

 The pinching effect that occurs in two-dimensional bonding problems, noted in Reports 2 and 4 above, is analyzed in some detail.
- 12. A. J. Durelli, V. J. Parks, H. C. Feng and F. Chiang, "Strains and Stresses in Matrices with Inserts,"-- May 1967.

 Stresses and strains along the interfaces, and near the fiber ends, for different fiber end configurations, are studied in detail.
- 13. A. J. Durelli, V. J. Parks and S. Uribe, "Optimization of a Slot End Configuration in a Finite Plate Subjected to Uniformly Distributed Load,"--June 1967.

 Two-dimensional photoelasticity was used to study various elliptical ends to a slot, and determine which would give the lowest stress concentration for a load normal to the slot length.
- 14. A. J. Durelli, V. J. Parks and Han-Chow Lee, "Stresses in a Split Cylinder Bonded to a Case and Subjected to Restrained Shrinkage,"--January 1968.

 A three-dimensional photoelastic study that describes a method and shows results for the stresses on the free boundaries and at the bonded interface of a solid propellant rocket.
- 15. A. J. Durelli, "Experimental Stress Analysis Activities in Selected European Laboratories"--August 1968.

 This report has been written following a trip conducted by the author through several European countries. A list is given of many of the laboratories doing important experimental stress analysis work and of the people interested in this kind of work. An attempt has been made to abstract the main characteristics of the methods used in some of the countries visited.

- 16. V. J. Parks, A. J. Durelli and L. Ferrer, "Constant Acceleration Stresses in a Composite Body"--October 1968.

 Use of the immersion analogy to determine gravitational stresses in two-dimensional bodies made of materials with different properties.
- 17. A. J. Durelli, J. A. Clark and A. Kochev, "Experimental Analysis of High Frequency Stress Waves in a Ring"--October 1968.

 A method for the complete experimental determination of dynamic stress distributions in a ring is demonstrated. Photoelastic data is supplemented by measurements with a capacitance gage used as a dynamic lateral extensometer.
- 18. J. A. Clark and A. J. Durelli, "A Modified Method of Holographic Interferometry for Static and Dynamic Photoelasticity"--April 1968.

 A simplified absolute retardation approach to photoelastic analysis is described. Dynamic isopachics are presented.
- 19. J. A. Clark and A. J. Durelli, "Photoelastic Analysis of Flexural Waves in a Bar"--May 1969.

 A complete direct, full-field optical determination of dynamic stress distribution is illustrated. The method is applied to the study of flexural waves propagating in a urethane rubber bar. Results are compared with approximate theories of flexural waves.
- 20. J. A. Clark and A. J. Durelli, "Optical Analysis of Vibrations in Continuous Media"--June 1969.

 Optical methods of vibration analysis are described which are independent of assumptions associated with theories of wave propagation. Methods are illustrated with studies of transverse waves in prestressed bars, snap loading of bars and motion of a fluid surrounding a vibrating bar.
- 21. V. J. Parks, A. J. Durelli, K. Chandrashekhara and T. L. Chen, "Stress Distribution Around a Circular Bar, with Flat and Spherical Ends, Embedded in a Matrix in a Triaxial Stress Field"--July 1969.

 A Three-dimensional photoelastic method to determine stresses in composite materials is applied to this basic shape. The analyses of models with different loads are combined to obtain stresses for the triaxial cases.
- 22. A. J. Durelli, V. J. Parks and L. Ferrer, "Stresses in Solid and Hollow Spheres Subjected to Gravity or to Normal Surface Tractions"-- October 1969.

 The method described in Report No. 10 above is applied to two specific problems. An approach is suggested to extend the solutions to a class of surface traction problems.
- 23. J. A. Clark and A. J. Durelli, "Separation of Additive and Subtractive Moiré Patterns"--December 1969. A spatial filtering technique for adding and subtracting images of several gratings is described and employed to determine the whole field of Cartesian shears and rigid rotations.

- 24. R. J. Sanford and A. J. Durelli, "Interpretation of Fringes in Stress-Holo-Interferometry"--July 1970.

 Errors associated with interpreting stress-holo-interferometry patterns as the superposition of isopachics (with half order fringe shifts) and isochromatics are analyzed theoretically and illustrated with computer generated holographic interference patterns.
- 25. J. A. Clark, A. J. Durelli and P. A. Laura, "On the Effect of Initial Stress on the Propagation of Flexural Waves in Elastic Rectangular Bars"--December 1970.

 Experimental analysis of the propagation of flexural waves in prismatic, elastic bars with and without prestressing. The effects of prestressing by axial tension, axial compression and pure bending are illustrated.
- A. J. Durelli and J. A. Clark, "Experimental Analysis of Stresses in a Buoy-Cable System Using a Birefringent Fluid"--February 1971.

 An extension of the method of photoviscous analysis is presented which permits quantitative studies of strains associated with steady state vibrations of immersed structures. The method is applied in an investigation of one form of behavior of buoy-cable systems loaded by the action of surface waves.
- 27. A. J. Durelli and T. L. Chen, "Displacements and Finite-Strain Fields in a Sphere Subjected to Large Deformations"--February 1972.

 Displacements and strains (ranging from 0.001 to 0.50) are determined in a polyurethane sphere subjected to several levels of diametral compression. A 500 lines-per-inch grating was embedded in a meridian plane of the sphere and moiré effect produced with a non-deformed master. The maximum applied vertical displacement reduced the diameter of the sphere by 27 per cent.
- A. J. Durelli and S. Machida, "Stresses and Strain in a Disk with Variable Modulus of Elasticity"--March 1972.

 A transparent material with variable modulus of elasticity has been manufactured that exhibits good photoelastic properties and can also be strain analyzed by moiré. The results obtained suggests that the stress distribution in the homogeneous disk. It also indicates that the strain fields in both cases are very different, but that it is possible, approximately, to obtain the stress field from the strain field using the value of E at every point, and Hooke's law.
- 29. A. J. Durelli and J. Buitrago, "State of Stress and Strain in A Rectangular Belt Pulled Over a Cylindrical Pulley"--June 1972.

 Two- and three-dimensional photoelasticity as well as electrical strain gages, dial gages and micrometers are used to determine the stress distribution in a belt-pulley system. Contact and tangential stress for various contact angles and friction coefficients are given.

- 30. T. L. Chen and A. J. Durelli, "Stress Field in a Sphere Subjected to Large Deformations"--June 1972.

 Strain fields obtained in a sphere subjected to large diametral compressions from a previous paper were converted into stress fields using two approaches. First, the concept of strain-energy function for an isotropic elastic body was used. Then the stress field was determined with the Hookean type natural stress-natural strain relation. The results so obtained were also compared.
- 31. A. J. Durelli, V. J. Parks and H. M. Hasseem, "Helices Under Load"-July 1973.

 Previous solutions for the case of close coiled helical springs and for
 helices made of thin bars are extended. The complete solution is
 presented in graphs for the use of designers. The theoretical development
 is correlated with experiments.
- 32. T. L. Chen and A. J. Durelli, "Displacements and Finite Strain Fields in a Hollow Sphere Subjected to Large Elastic Deformations"--September 1973. The same methods described in No. 27, were applied to a hollow sphere with an inner diameter one half the outer diameter. The hollow sphere was loaded up to a strain of 30 per cent on the meridian plane and a reduction of the diameter by 20 per cent.
- A. J. Durelli, H. H. Hasseem and V. J. Parks, "New Experimental Method in Three-Dimensional Elastostatics"--December 1973.

 A new material is reported which is unique among three-dimensional stress-freezing materials, in that, in its heated (or rubbery) state it has a Poisson's ratio which is appreciably lower than 0.5. For a loaded model, made of this material, the unique property allows the direct determination of stresses from strain measurements taken at interior points in the model.
- 34. J. Wolak and V. J. Parks, "Evaluation of Large Strains in Industrial Applications"--April 1974.

 It was shown that Mohr's circle permits the transformation of strain from one axis of reference to another, irrespective of the magnitude of the strain, and leads to the evaluation of the principal strain components from the measurement of direct strain in three directions.
- 35. A. J. Durelli, "Experimental Stress Analysis Activities in Selected European Laboratories"--April 1975.

 Continuation of Report No. 15 after a visit to Belgium, Holland, Germany, France, Turkey, England and Scotland.
- 36. A. J. Durelli, V. J. Parks and J. O. Bühler-Vidal, "Linear and Non-linear Elastic and Plastic Strains in a Plate with a Big Hole Loaded Axially in its Plane"--July 1975.

 Strain analysis of the ligament of a plate with a big hole indicates that both geometric and material non-linearity may take place. The strain concentration factor was found to vary from 1 to 2 depending on the level of deformation.

- 37. A. J. Durelli, V. Pavlin, J. O. Bühler-Vidal and G. Ome, "Elastostatics of a Cubic Box Subjected to Concentrated Loads"--August 1975.

 Analysis of experimental strain, stress and deflection of a cubic box subjected to concentrated loads applied at the center of two opposite faces. The ratio between the inside span and the wall thickness was varied between approximately 5 and 121.
- 38. A. J. Durelli, V. J. Parks and J. O. Bühler-Vidal, "Elastostatics of Cubic Boxes Subjected to Pressure"--March 1976.

 Experimental analysis of strain, stress and deflections in a cubic box subjected to either internal or external pressure. Inside span-to-wall thickness ratio varied from 5 to 14.

Introduction

A number of important two-dimensional problems in technology involves sinusoidally varying stresses. Problems of this type can be studied photoelastically using a synchronized stroboscope or a high-speed movie camera. This brief paper presents an alternative method for studying cyclic stresses. The method requires no special equipment—merely the standard apparatus commonly used in photoelasticity laboratories; namely, a polariscope and a still camera.

The time-average technique was first utilized in holography to study vibrations by Powell and Stetson. (1) They showed that when a hologram was made of an object undergoing steady oscillation, its reconstruction exhibited time-averaged interference patterns superimposed on the object image. These patterns could be described mathematically by a zeroth order Bessel function containing the vibrational amplitudes in its argument. The same concept was also applied to moiré methods, (2,3) and in speckle interferometry (4-7) to determine either displacements or displacement-derivatives due to vibrations.

The Method

A photoelastic model undergoing steady-state sinusoidally cyclic loading is placed in a plane polariscope, or in a circular polariscope set for either dark or light field. The time varying isochromatics are photographed by a camera with an exposure time equal to one or several periods of the loading cycle. The processed photograph will exhibit a time-averaged isochromatic fringe pattern which depicts the amplitude of the cyclic stresses, and the fringe orders correspond to the roots of a Bessel function of zeroth order. In the case of the linear polarization arrangement, isoclinics are also recorded.

Time-averaged cyclic stress-optic law

Assuming that the polariscope is in a dark field arrangement with circular polarization, it is well-known (8) that the received intensity by the camera can be represented by:

$$I_{D} = 1 - \cos 2 D \tag{1}$$

where D = $\frac{\pi d}{\lambda}$ (C₁ - C₂)(σ_1 - σ_2), d is the model thickness, C₁, C₂ are Maxwell stress-optics coefficients, λ is the wavelength of light and σ_1 , σ_2 are the principal stresses.

Since in the case considered the model is undergoing sinusoidally varying stresses, $(\sigma_1 - \sigma_2)$ is time dependent and can be expressed as

$$\frac{\sigma_1 - \sigma_2}{2} = \tau_0 \cos(\omega t + \phi) \tag{2}$$

where

 τ_o = the maximum shear stress

 ω = circular frequency of the loading cycle

If a photographic film is exposed to the intensity shown in Eq. (1) with an exposure time equal to k periods, the light energy received is the integration of $I_{\overline{D}}$ over the exposure time, given by:

$$E_{D} = \int_{0}^{kT} I_{D} dt$$
 (3)

where $E_{\overline{D}}$ is the exposure and T is the period of the loading cycle.

Integration of Eq. (3) yields:

$$E_{D} = kT\left\{1 - J_{O}\left[\frac{4\pi dC}{\lambda} \tau_{O}\right]\right\}$$
 (4)

where $C = C_1 - C_2$

The above equation predicts that the formation of time-averaged isochromatics with minimum intensity (dark fringe) occurs when the zeroth order Bessel function attains its maximum value. The fringes depict the distribution of τ_o , the amplitude of the cycling stresses.

A similar analysis will show that the time-averaged isochromatic fringes in a light field polariscope can be represented by

$$E_{L} = kT\left\{1 + J_{o}\left[\frac{4\pi dC}{\lambda} \tau_{o}\right]\right\}$$
 (5)

Here, dark fringes correspond to the minimum values of J_0 .

In the case of the plane polariscope, the time integrated intensity received on the photographic film can be shown to be

$$E = kT \left\{ 1 - J_0 \left[\frac{4\pi dC}{\lambda} \tau_0 \right] \right\} \cdot \sin 2\theta$$
 (6)

Where θ is the isoclinic angle.

Fringe Interpretation

In instantaneous photoelasticity, the relationship between the maximum shear stress τ and dark fringes is given as:

$$\tau = n \frac{f}{d} \tag{7}$$

where n = fringe order and $f(=\frac{\lambda}{C})$ is the material fringe value.

In a dark field circular polariscope, n takes the values: 0, 1, 2, 3, and in the light field circular polariscope the values: $\frac{1}{2}$, $\frac{3}{2}$, $\frac{5}{2}$,

Note that in the time-average method presented in this paper, fringe pattern is described by Eqs. (4) and (5). Thus the dark fringes occur when the Bessel function is maximum in a dark field arrangement and minimum in a light field setup. When Eq. (7) is used to interpret time-averaged fringes, the values of n should be

$$n = 0, 1.12, 2.12, 3.12, 4.12, \dots$$

for dark field polariscope, and

$$n = 0.610, 1.62, 2.62, 3.62, 4.62, \dots$$

for light field.

Higher fringe orders can be found in a table of zeroth order Bessel functions.

Experimental Demonstration

Chosen for demonstration were two cantilever beams, one rectangular, and the other with a notch. Both beams were clamped at one end and periodically loaded at the free end. The sinusoidally varying loading was produced by hanging a spring-mass system from the free end. When the mass was set to free vibration (damping considered negligible), a sinusoidally varying load was applied to the beam. In the experiments, the time varying isochromatics were photographed with an exposure time of approximately two periods of the oscillation. The processed photographs

exhibited the time-averaged isochromatics shown in Figs. 1(a) and 2(a) for beams with and without a notch, respectively. For the purpose of comparison, isochromatics of the rectangular beam, under static loading of same free end deflection as the amplitude of the cyclic loading, are shown in Figs. 1(b) and 2(b). A comparison of the reading of the time-averaged and static isochromatic fringe orders are also presented in Figs. 1 and 2.

This illustration shows application of the method to a very simple case for which the static solution practically coincides with the static one. This method, as easily visualized, can be applied to any two-dimensional problem of in-plane complex system of vibrations.

Conclusion

It has been shown that time-averaged isochromatics fringes may be recorded when a photoelastic model is undergoing a sinusoidally varying stress. The fringe pattern is depicted by a Bessel function of zero-order since the amplitude of Bessel function decreases with increasing argument value. The fringe visibility diminishes as the fringe order increases. This may be a problem in cases involving high stress concentrations. This is already evidenced by the time-averaged isochromatics of the beam with a notch.

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References

- 1. Powell, R. L. and Stetson, K. A., "Interferometric Vibration Analysis by Wavefront Reconstruction," J. Opt. Soc. Am. 55, 1593, 1965.
- Hovanesian, J. D. and Hung, Y. Y., "Moiré Contour-sum, Contour-Difference and Vibrational Analysis of Arbitrary Objects," <u>Applied</u>
 Optics, Vol. 10, No. 12, 1971.
- Vest, C. M. and Sweeney, D. W., "Measurement of Vibrational Amplitude by Modulation of Projected Fringes," <u>Applied Optics</u>, Vol. 11, No. 2, 449, 1972.
- 4. Tiziani, H. J., "Application of Speckling for In-Plane Vibration Analysis," Optica Acta, 18, 1971.
- 5. Hung, Y. Y. and Taylor, C. E., "Speckle-Shearing Interferometric

 Camera--A Tool for Measurement of Derivatives of Surface Displacements,"

 Proceedings of the Society of Photo-Optical Instrumentation Engineers,

 Vol. 41, 169-175, 1973.
- 6. Archbold, E. and Ennos, A. E., "Two-dimensional Vibrational Analysis by Speckle Photography," Optics and Laser Technology, Feb. 1975.
- 7. Hung, Y. Y., Hovanesian, J. D. and Durelli, A. J., "New Optical Method to Determine Vibration-Induced Strains with Variable Sensitivity after Recording," to be published in J. of Applied Mechanics.
- 8. Durelli, A. J. and Riley, W. F., "Introduction to Photomechanics, Prentice-Hall, p. 34, 1965.

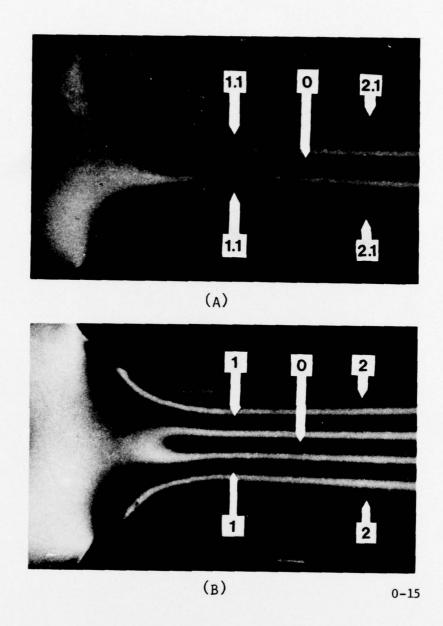


Figure 1 Dark Field Isochromatics in a Cantilever Beam Subjected to: (a) Time-Averaged Steady Vibration and (b) Steady Loading Applied to the Free End (Same Amplitude in both Cases at the Free End)

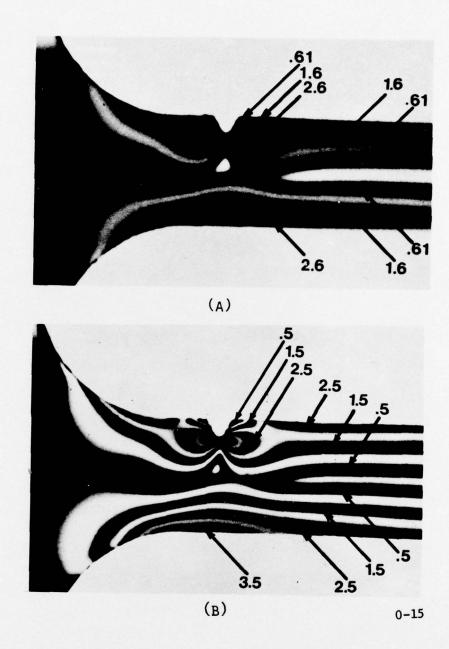


Figure 2 Light Field Isochromatics in a Cantilever Beam Subjected to: (a) time-Averaged Steady Vibration and (b) Steady Loading Applied to the Free End (Same Amplitude in both Cases at the Free End)

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This paper describes an experimental method	nod whereby the amplitude of
cyclic stresses may be readily determined by	time-averaged photoelasticity.
Using an ordinary polariscope with a monochron	matic light source, "time-
averaged isochromatics fringes are formed if	the photographic film in the
camera is exposed with an exposure time equal	ond state cyclic location
while the photoelastic model is undergoing sto	
The fringe pattern depicts amplitudes of the	oscillating stresses according

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Unclassified SECURITY CLASSIFICATION OF THIS PAGE(When Data Entered) to the zeroth order Bessel function. These properties permit the determination of a time-averaged cyclic stress-optic law. It is also possible to use the method to determine time-averaged isoclinics. The method has great potentiality in the study of in-plane vibrations.